JOURNAL OF MATERIALS SCIENCE **32** (1997) 4731–4736

Limitations of pore-stress concentrations on the mechanical properties of porous materials

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Severe limitations on pore-stress concentration effects on mechanical properties are shown. First, the porosity dependence of materials with dilute porosities is not consistent with significant variations in stress concentrations with the stress state. Second, in non-dilute porosities, pore stress concentration effects are reduced due to pore-stress interactions as pore spacings decrease. Such reduction of stress concentrations is seen as supporting the concept of the minimum solid area correlating with properties in porous materials, and the similarity between the porosity dependence of mechanical properties and electrical and thermal conductivity. Finally, crack-pore interactions often limit the effects of pore-stress concentrations, e.g. due to small pore sizes. However, some effects of pore-stress concentrations may occur due to tensile failure from a few or an isolated pore, or more general porosity under compressive loading, but even in these cases pore shape-stress concentration effects are significantly mitigated.

1. Introduction

Many models of the porosity dependence of mechanical properties directly or indirectly invoke the effects of stress concentrations from pores as a key factor in the mechanical behaviour of porous, brittle solids. [1–4]. Commonly, the maximum tensile stress concentration is cited as the key factor. However, the more detailed analysis presented in this note, and elsewhere [5] shows that stress concentrations from pores commonly play, at best, a limited role in defining the mechanical behaviour of a porous solid. The primary exceptions to this are in actual mechanical failure. However, even in this case the role of stress concentrations is muted. Three sets of arguments are presented to show these limitations: (1) the variable nature of stress concentrations for isolated pores, (2) the interactive nature of stress concentrations as porosity increases so pores are no longer isolated, and (3) interaction of pores and cracks.

2. Evaluation

2.1. Isolated pores – dilute porosity

Consider first dilute concentrations of pores, where the stress concentrations of individual pores are clearly defined. Such concentrations extensively vary in their limits with pore shape and the orientation of such shapes relative to the stress axis, as well as around the pore. They also vary with the nature of the stress and with the proximity of the pore to the surface (hence also introducing some pore size-specimen size effects). Despite concerns about stress concentrations being important in the effects of porosity on mechanical properties, apparently no thorough study of this problem has been made. However, some data for different stresses, e.g., uniaxial tension or compression and uniaxial and biaxial flexure exist. A comparison of such data for various stress states shows no clear evidence of stress concentration effects on mechanical behaviour.

Consideration of cylindrical and spherical pores, which are the simplest pore shapes commonly encountered and widely analysed, clearly show the limitations of stress concentration effects on mechanical properties. A summary of the limiting stress concentrations for such pores is shown in Table 1. That stress concentrations do not determine the mechanical behaviour of porous materials is clearly shown by considering cylindrical pores. Such pores (and in fact any prismatically shaped pores) oriented with the axes of cylindrical pores aligned parallel to the stress axis have no stress concentrations, whether the uniaxial stress is tensile or compressive. However, both the elastic and strength (as well as other, e.g. thermal and electrical) properties of such bodies are reduced by such porosity despite there being no stress concentrations. Such reductions are instead consistent with minimum solid cross-sectional areas (which, for measurement parallel to the axis of aligned cylindrical pores are also the average solid cross-sectional areas) normal to the pore-stress axis.

The lack of a correlation between stress concentration and mechanical behaviour of pores is further highlighted by considering cylindrical pores normal to the stress axis or spherical pores relative to any stress axis. Even spherical pores have stress concentrations which vary with the nature of the stress (Table 1). Thus, the stress concentrations decrease when going from homogeneous uniaxial tension, to tension from bending, to homogeneous uniaxial compression. In

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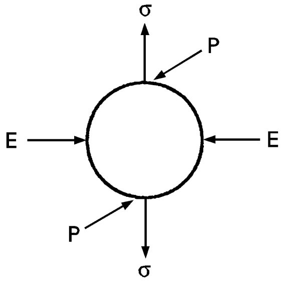
TABLE I Maximum stresses[a] on spherical or cylindrical holes[b]

Applied stress	Spherical hole[b]	Cylindrical hole(b,c)
Uniaxial tension	~ 2(E)	3(E); 1(P)
Uniaxial bending		2
Uniaxial torsion		
Uniaxial compression	-1(P)	-1(P)
Biaxial tension		2(P), 2(E)
Biaxial tension/compression		4(P), 4(E)
Triaxial tension	$\frac{3}{2}$	

[a] Stresses are shown as multiples of the applied stress; a factor of 1 means the peak stress is the applied stress and factors > 1 mean a stress concentration. A negative sign means a reversal of the stress; i.e., from compressive to tensile or *vice versa*.

[b] For cylindrical pores oriented perpendicular to the stress axis (or axes). There is no stress concentration for such pores parallel to the stress axis.

[c] Locations of the maximum stresses are shown for both types of pores as shown below; i.e., P for Polar and E for equatorial for a spherical pore. The same designations are used for a cylindrical pore except the poles are now lines and not points and the "equator" two lines rather than a circle.



fact, there is no concentration of the compressive stress for spherical pores, only the occurrence of a localized tensile stress whose maximum equals the value of the applied compressive stress. If maximum stress concentrations were an important factor in mechanical properties, the stress dependence of such stress concentrations would imply that the porosity dependence of all mechanical properties would be dependent on the stress state of the body. There are obvious and well known dependences of the mechanical behaviour of brittle materials on stress state for dense materials that will carry over to porous materials, e.g., differences between biaxial and uniaxial tension, and especially those from uniaxial compression-failure stresses. However, such dependences of mechanical behaviour on the stress state show no relationship to the mechanical dependence of porosity. Thus, for example, there is no evidence of the elastic properties of porous bodies depending on the stress state. Reported correlations of elastic properties of ceramics with the maximum stress concentration of spheroidal pores has been shown to be similar to, but substantially less accurate than, that with minimum solid area [5]. Further, while compressive strengths are typically much higher than tensile strengths for both dense and porous brittle materials, their porosity dependence does not correlate with pore stress concentrations. An earlier survey [1] had indicated a possible greater porosity dependence of ceramics for properties based on compressive loading (i.e., compressive strength, hardness, and wear) than for tensile loading. However, the data base for this was limited, and possible fabrication-pore structure effects recently identified [6] were not evaluated. More fundamental to the present evaluation, this earlier indicated higher porosity dependence in compressive loading is contrary to a correlation with stress concentration since the latter is lower in compression than tension.

2.2. Pore interactions – non-dilute porosity

Pore interactions begin to no longer be negligible when the centre-to-centre distance between two pores is ~ 2 times their diameter (i.e., their surface to surface spacing is ~ 1 diameter). Thus, consider simple cubic packing of identical spherical voids. Such packing (which has nearly identical density and porosity effects as random packing) translates to simple cubic stacking of identical solid cubes with identical, centred bubbles in them (Fig. 1). The porosity of such a body is thus the volume fraction porosity (P) of a unit cubical cell; i.e.,

$$P = \frac{4\pi}{3}\left(\frac{r}{l}\right)^3 \quad (1)$$

where r is the pore radius and l is the cube edge. Thus, when spherical pores are separated (surface to surface) by one diameter $l = 4r$, which yields $P = \pi/48 \sim 0.065$, i.e., at $< 7\%$ porosity.* Clearly, pores between packed particles (Fig. 1) have the same or closer spacing at comparable porosity.

Interactions between pores thus can begin to be of concern as low as 5–10% porosity. The addition of

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* Note that the cylindrical pores, considered later, have centre-to-centre separations of twice their diameter at $P \sim 20\%$, i.e., nearly three times higher than for spherical pores. However, higher stress concentrations of cylindrical pores means that they will begin to interact at greater spacings than spherical pores.

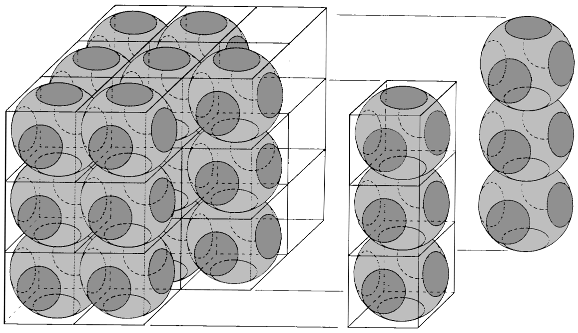

Figure 1 Schematic for modelling porosity effects on materials as well as effects of the interaction of stress concentrations from pores. Cubes stacked in simple cubic packing are illustrated to model either of the two basic types of porosity, namely those pores that form between partially bonded (e.g. sintered) spherical particles (far right), or for spherical voids (e.g. bubbles) which are defined by surrounding solid material in the form of a cubic outline (middle right). (Such simple cubic packing is a good representation, both in porosity values and in resultant property effects, for random packing of uniform pores.)

more pores increases the interaction with an associated reduction in spacing between pores. Such interactions have been studied in detail for two pores. Thus, for example, the peak stress concentration for two cylindrical holes in a plate stressed parallel with the axis through both hole centres is simply that of a single hole (3, at their equator) when the two holes have merged into one (i.e., O centre-to-centre spacing) [8]. For stressing the two holes normal to their centre-to-centre line, the peak stress concentration is ~ 3.9 (at their equators) when the holes just touch ($P = \pi/4 \sim 0.784$ for simple cubic packing of cylindrical holes. The corresponding P value for spherical holes is $\pi/6 \sim 0.524$, i.e., where porosity commences to be interconnected, hence open.) However, the stress concentration drops significantly as the two cylindrical holes merge into a lenticular pore of decreasing lens thickness. Partial solutions are also available for arrays of cylindrical holes in plates, showing increased stress when the holes just touch, but dropping off rapidly otherwise [8]. Again, the above stress concentrations are lower under biaxial versus uniaxial loading [7, 8]. More detailed considerations show that most stress concentrations decrease with increasing numbers of pores with decreasing spacings.

A reasonable way of evaluating in more detail the effects of the interaction of pore-stress concentrations in bodies with non-dilute porosity is indicated by the model systems in Fig. 1. There, the two basic ways of generating porosity, namely introducing bubbles (or fugative bodies) or partial bonding of particles (e.g., sintering) are shown respectively as simple cubic stacking of either cubical blocks of solids with identical spherical pores within them, or of stacked spherical particles that are partially bonded together. (Again, note that simple cubic packing of particles is a good approximation of random packing [9]. Thus, as at least a first order, and probably a fairly good, approximation, one could model partially-bonded equivalent sized spherical particles as a single shaft with multiple grooves, the grooves having a centre-to-centre spacing equal to the particle diameter (with a root radius given by $\sim (x^2/2R)$ or $(x^2/4R)$, respectively for sintering via vapour phase versus diffusion processes, at least for $(x/R) < 0.3$, where x is the neck radius, and R the particle radius) [10]. The other basic model for evaluating interactive stress concentration–porosity effects would be that obtained by stacking cubes with bubbles in them (Fig. 1). This can to a first, but presumably a reasonable, approximation, be modelled as a square cross-section rod with a series of in-line, equal sized spherical bubbles down the centre of the rod with the spacings between the bubbles being given by Equation 1. Unfortunately, such stress solutions for either of these idealized but applicable pore structures do not appear to exist in the literature. However, there are two types of approximations which illustrate the overall effects of reduced stress concentration effects with increasing pore interactions. These are based on solutions to bodies with cylindrical holes in them or grooves, e.g., straight grooves, in the surface of a strip of material (with a semicircular groove radius, Fig. 2, which also shows effects of hole or groove dimensions relative to the overall body dimensions, as well as showing less stress concentration from bending than uniform uniaxial tension [11].

Consider next the effect of a chain of equal size cylindrical holes. Fig. 3 shows the stress concentrations for such a series, i.e., chain of holes (pores) where there is a uniaxial stress parallel with the chain of holes, or a biaxial stress on them [12]. Both clearly show that the stress concentration diminishes significantly as the holes get closer and closer together. (Also, note lower net stress concentrations, even in the limit, for such holes under biaxial versus uniaxial stress.) In fact, note that the stress concentrations become quite low (in fact, disappear, i.e., becoming one) in the above two respective cases at the point where the holes touch, i.e., where their centre-to-centre spacing is equal to their diameter. For cylindrical holes, this occurs at a porosity of 78.5%. Note that

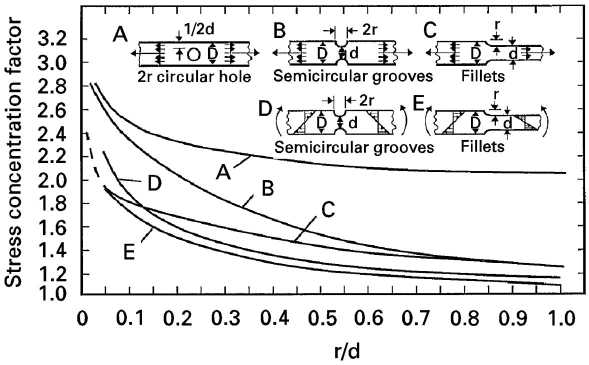

Figure 2 Summary of stress concentration factors in both uniaxial tension and in bending for strips of materials containing either a cylindrical hole, cylindrical grooves, or fillets (the latter being either the limit of a surface groove which has been separated into two surface steps, or as the limit of the large number of grooves in the surface). Note the more severe stress concentration for the circular hole versus the groove versus the fillet, and higher stress concentrations in uniaxial tension versus bending for the same geometries, but also hole-to-specimen size effects. Combined data from Peterson's compilation [12].

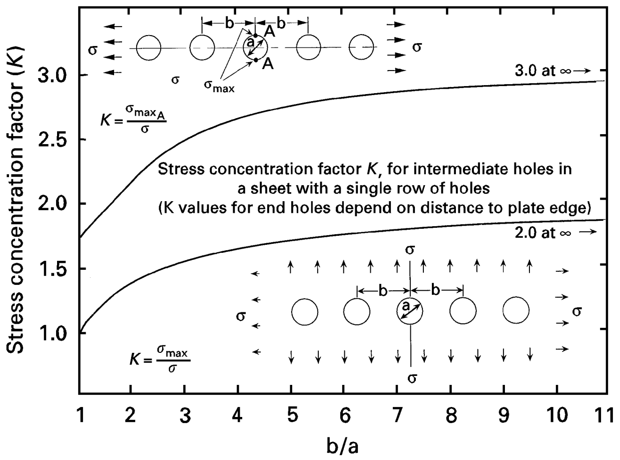

Figure 3 Stress concentration factors for a chain of holes in either uniaxial tension (parallel with the chain length) or in biaxial tension. Note the higher stress concentration in uniaxial tension and this being in the limit the same as for a single hole, and lower stress concentrations in biaxial tension. Combined data from Peterson's compilation [12].

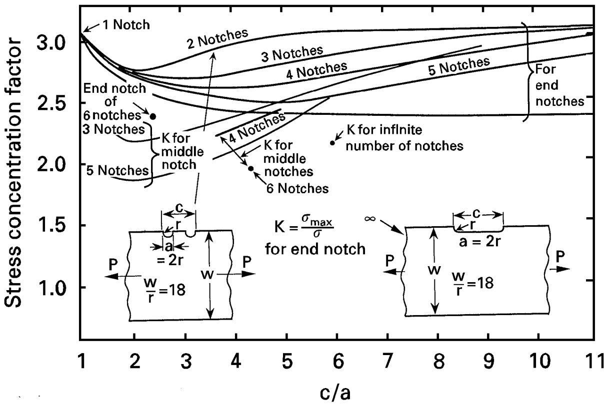

Figure 4 Stress concentration factors for multiple semi-circular notches in the surface of a strip of material. Note that stress concentrations for both the end notches for the increasing number of notches are shown as well as for the middle notches. Combined data from Peterson's compilation [12].

there are also some solutions in the literature for two chains of holes, including varying separations and juxtapositions of the two chains of holes [12]. These do not significantly change the above results.

Next, consider the other basic system for evaluating the effects of interactive stress concentrations, in this case as an approximation for a chain of sintered particles. This is done by considering a rectangular strip with increasing numbers and decreasing spacing between surface grooves. Again, while there are varying solutions in the literature for various shaped grooves, for simplicity, cylindrical shaped grooves will be considered. It should be noted that there are also solutions in the literature for semi-circular and various other grooves in circular shafts. Unfortunately, no solutions were found for multiple grooves along uniform circular shafts, which would have been directly applicable to the model of interest here. Fig. 4 summarizes some of the pertinent literature solutions [12]. Note that if there is a finite number of notches, that the stress concentration on the end notches is always substantially higher than on the intermediate notches. However, in either case the net stress concentration decreases as the number of notches increases and, in fact, approaches the stress concentration for a fillet (see also Fig. 2). Even more pertinent to the present discussion is a significantly lower level of the stress concentration for the notches in the middle of a group of notches and the fact that these also continue to decrease in level as the number of notches increases.

Thus, the examination of literature solutions approximating the two porosity models shown in Fig. 1 shows that as resultant pores become more interactive one generally sees a significant reduction in the effect of individual and absolute levels of stress concentrations which, as noted in the previous section, are of questionable impact on mechanical properties in even isolated pore cases. This diminution of the effects of stress concentrations as they become more closely spaced is a generally recognized effect which is schematically shown in Fig. 5 (a–e). Basically, the result is that an isolated stress concentrator has significant stress gradients around it. However, due to the reduced load carrying capability in the immediate vicinity of the stress concentrator, as one brings more and more stress concentrations into interaction, there is less and less load to be transmitted between adjacent stress concentrators. Thus, in the limit, for the example of surface stress concentrators, the body simply begins to act like one whose exterior dimensions were simply reduced by the depths of the stress concentrators. In the case of a series or chain of aligned pores, as in Fig. 1, this implies that the thin webs between the pore chains carry the load, which is consistent with minimum solid area models [1]. Finally, while Fig. 5 (a–e) shows schematically the lines of stress concentration, this same diagram would also be applicable if one were considering flow paths for heat or electrical conductivity, thus indicating identical results whether one was dealing with stress or such conductivity effects. This is also consistent with experimental data for the porosity dependence of thermal and electrical conductivities being the same as for mechanical properties for the same pore structures.

2.3. Pore–crack effects

In most tensile loading cases, cracks determining strength and especially those used in determining the stress concentration factor K_{IC}, are large in comparison to the pores. Thus, any stress concentrations from the pores will be very local perturbations of the crack tip stress. Further, stress concentrations associated with pores typically entail both tensile and compressive stress, which would further reduce the effect of their stress concentrations, since cracks would experience mixtures of such stress concentrations from smaller pores along or near the periphery of such cracks. Thus, stress concentrations from pores small in comparison with cracks should not be significant.

As the pore and crack sizes begin to approach one another, pore shape and possibly stress concentration

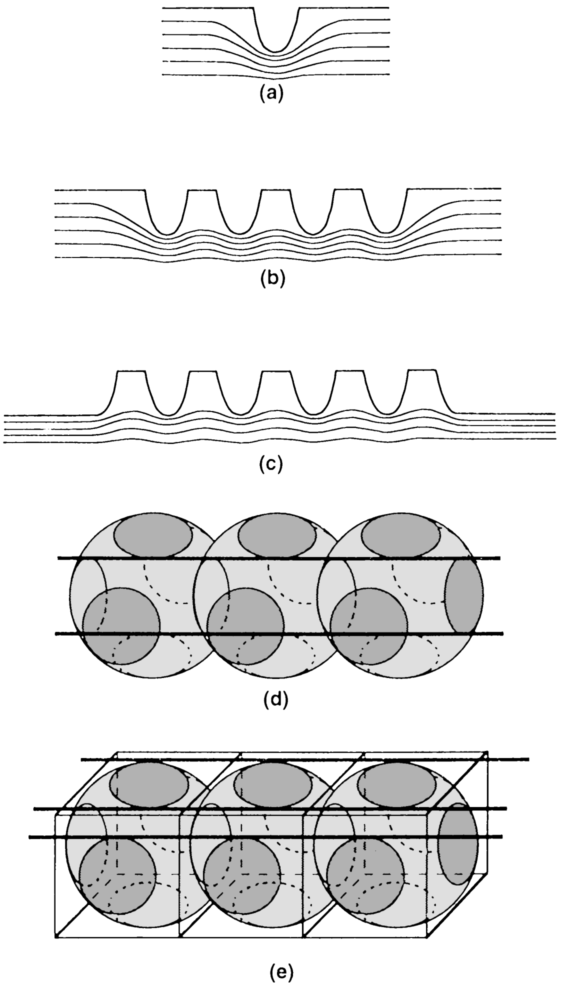

Figure 5 Schematic illustration of the effects of increasing numbers of surface notches on crest profiles. Note that increasing the number of notches and decreasing their spacing (a–c) basically results in a body with surface notches behaving like one which has been simply reduced in thickness or diameter by the depth of the notches, as shown by Peterson [12]. This implies that the major load carrying ligaments for the model of simple cubically stacked particles or bubbles from Fig. 1 are the ligaments whose transverse dimensions are those of the aligned bond areas (d) and aligned minimum web thicknesses (e); i.e., the ligaments just encompassing the minimum solid bond areas normal to the measuring direction. Note also that the stress profiles of (a–c) would also reflect paths of transport for thermal conductivity or electrical conductivity along the same ligaments of (d) and (e).

effects may become more significant. However, when a few or several pores link together to cause tensile failure, any effects due to pore shape and especially stress concentrations are still considerably mitigated due to the variations in the pores and the crack linkages between them. Even when a single pore is the source of the tensile failure, shape and especially stress concentration effects, are limited. First, the dominant factor in such pore initiated failures is the size of the pore in a plane approximately normal to the tensile stress, which is primarily a function of pore size and secondarily of pore shape and orientation relative to the stress axis [1]. Such large pore origins will often be from atypical (i.e., isolated large) pores which do not necessarily have any relation to the average pore shape or stress concentrations. Further, failure from even a single pore commonly entails its combination with a machining flaw, peripheral crack, or both [13–20]. The presence of such cracks would commonly be dominant over any pore shape–stress concentration effects since the stress concentration from the crack will exceed that of the pore. Also, such cracks will commonly extend a substantial distance around the pore, hence often crossing varying levels of pore stress concentration. Thus, pore shape–stress concentration effects are likely to be quite limited under tensile loading.

Theoretically, under compressive loading, there should be more opportunity for effects of pore shape and stress concentrations than in tensile loading. This arises since crack propagation is inhibited by compressive stresses, such that under compressive loading crack generation and propagation occur primarily due to local tensile stress from pores or other heterogeneities. Crack initiation from several isolated pores resulting in multiple fracturing of specimens in compressive loading has been shown experimentally [21]. With increasing numbers of pores there is increasing interaction of the local stresses and local resultant cracks. A key result of this is that compressive failure of brittle materials is typically the result of cumulative damage from a number of crack generation–linkage-propagation events rather than a single weak link source as for tensile failure [21–24]. Thus, stress concentrations from pores (as well as cracks and inclusions) are seen as sources of crack generation or propagation in compressive loading. However, this increasingly leads to an averaging of pore stress concentration effects, e.g., due to pore shape–stress orientation effects, as well as possible effects of slip and twinning as porosity increases. Again, such compressive effects are also limited by the lower maximum stress concentration in compression than tension (Table I). Slip or twinning are also more likely to be activated by the higher levels of compressive strengths, [22] while again possibly influenced by pore shape–stress concentration effects, there are also mitigating effects of such concentrations on cracking. These include reducing or redistributing stress, as well as effects of pore shape variations along with their orientation relative to the principal stress, their relation to nearby pores, and again lower maximum stress concentrations in compression than tension.

3. Summary and conclusions

Arguments based on property trends versus porosity and model analytical solutions for stress concentrations of notches and holes in the literature are cited to show that the frequent assertion that stress concentrations, especially the maxima, from pores are a dominant factor in pore effects on mechanical properties must be questioned. Thus, (1) the porosity dependence of materials with dilute porosities is not consistent with significant variations in maximum stress concentrations with the stress state, (2) in non-dilute porosities, pore stress concentration effects are often substantially reduced due to pore-stress interactions as pore spacings decrease, and (3) in pore-crack combinations stress concentrations are commonly dominated by the crack, not the pore. Stress concentration effects are significantly more complex and muted than commonly implied, but their collective effects may be approximated by minimum load bearing areas.

Acknowledgements

Professor J. R. Rice of Harvard University is thanked for initial discussions of the effects of increasing numbers and proximity of stress concentrations, e.g. as reflected in Fig. 5 (a–e) and initial guidance on references.

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Received 19 October 1994
and accepted 8 September 1995